Current Issues in Education
Mary Lou Fulton Teachers College • Arizona State University
PO Box 37100, Phoenix, AZ 85069, USA

Volume 19, Issue 2 | August 15, 2016 | ISSN 1099-839X

Professional Development for Mathematics Teachers: Using Task Design and Analysis

Hea-Jin Lee
Ohio State University at Lima

S. Asli Özgün-Koca
Wayne State University

Abstract:
This study is based on a Task Design and Analysis activity from a year-long professional development program. The activity was designed to increase teacher growth in several areas, including knowledge of mathematics, understanding of students' cognitive activity, knowledge of good questions, and ability to develop and improve high quality tasks. The study collected data from 30 classroom teachers. Results include teachers' initial understanding of the task, teachers' interpretations and awareness of the strategies their students used to solve the task, and the pattern of revised tasks. The process and findings of the study will expand teacher educators' understanding of structured approaches to link an inservice PD with teachers' work context as well as develop a better analytic framework within which teachers and teacher educators analyze student work.

Keywords: Task design and analysis, professional development, teacher growth

APA-Style Citation:
Lee, H.-J., & Özgün-Koca, S. A. (2016). Professional development for mathematics teachers: Using task design and analysis. *Current Issues in Education*, *19*(2). Retrieved from http://cie.asu.edu/ojs/index.php/cieatasu/article/view/1519

Accepted: May 3rd, 2016

An integral component of instruction is the notion of a *task*. A mathematical task is a set of problems or a single complex problem that focuses students' attention on a particular mathematical idea (Stein, Grover, & Henningsen, 1996). Mathematical tasks are central to student learning and require various levels of cognitive demand (Henningsen & Stein, 1997). The content and context of tasks influence the way students think and how they understand mathematics.

Beside the benefit to students, a task serves to inform the teacher's instruction. The quality of a task and how the task is implemented have a great influence on how a student learns mathematics (Krebs, 2005; Thompson, Carlson, & Silverman, 2007). In order to develop a student's active reasoning process and high-level thinking skills, the classroom environment should provide every student with opportunities to engage in rich mathematical tasks. The strategies used by students need to be understood in order to facilitate the mathematical discourse about these strategies, and to help students build on their mathematical knowledge and develop new strategies (Kazemi & Franke, 2004; Meyer, 1997). Therefore, it is critical that the teacher be capable of designing quality tasks.

This study is based on a professional development (PD) activity in task design and analysis. The activity was included as part of a year-long professional development program, aimed to improve a teacher's knowledge and skills needed for designing high quality mathematics tasks. The study has two goals: (1) to develop a conceptual framework for a PD program that uses student work to enhance a teacher's professional growth; and (2) to provide an analytic framework within which teachers and teacher educators analyze student work. The work of both students and teachers was analyzed in order to meet these goals and to investigate: teacher knowledge of mathematics, teachers' understanding of students' cognitive activity, and a teacher's ability to develop high quality tasks.

The Theoretical Framework

The theoretical framework of the study (Lee & Özgün-Koca, 2013, 2014) is based on the steps of the *Progression of Mathematical Tasks* (Stein et al., 1996) and the relationships among various task-related variables and students' learning outcomes (Henningsen & Stein, 1997). This is carried out using student work in teacher education (Kazemi & Franke, 2004), the *Life Cycle of Tasks* (Namiki & Shimizu, 2012), and *Professional Noticing of Children's Mathematical Thinking* (Jacobs, Lamb, & Philipp, 2010).

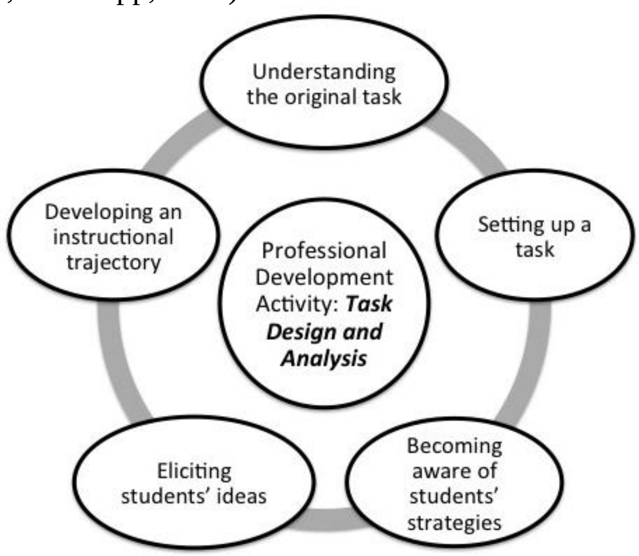


Figure 1. Theoretical framework of the study.

As seen in Figure 1, the teacher participates in five phases, their role transforming from student, instructor, task analyzer, to task designer.

- *Phase 1: Initial participation; understanding the original task*: In this phase, participating teachers interpret the intention of the original task and predict multiple solution strategies that students might use. For teacher educators, this phase may be used to evaluate a teacher's content knowledge and develop an appropriate PD plan to deepen it.

- *Phase 2: Setting up a task*: This is an onsite activity lead by teachers. In this phase, teachers collect students' solutions and "encourage students to use more than one strategy, to use multiple representations, and to supply explanations and justifications." (Henningsen & Stein, 1997, p. 529)
- *Phase 3: Becoming aware of student strategies:* Teachers analyze students' solution strategies and develop an understanding of students' cognitive activity. Teachers might use a commercially designed rubric or a self-designed rubric to create an official report of student work. For the purpose of the PD, teachers share sample student solutions and their analyses of the solutions with other teachers. Comparing students' solutions from different grade levels and different demographics can expand a teacher's understanding of a student's learning progress, as well as possible factors that impact the student's cognitive process.
- *Phase 4: Eliciting student ideas*: In this phase, teachers develop appropriate questions for each student in order to respond to the student's needs and to improve their learning outcomes. As part of the PD, teachers can follow up on each student's solutions by posing several thought-provoking questions. These questions should aim to enhance the student's understanding of the mathematical concept presented in the task, to encourage the student to examine their own cognitive process, to provoke higher level thinking, and, eventually, to improve the student's learning outcomes.
- *Phase 5: Developing an instructional trajectory*: This final phase is also the beginning of a new task design and analysis cycle. Teachers in this phase reflect individually on the entire cycle of the Task Design and Analysis activity. They revise the original task to better respond to their students' needs and improve their readiness.

Jacobs et al. (2010) introduced a framework named *Professional Noticing of Children's Mathematical Thinking*, which concentrates on the interweaving and interacting skills of: (a) attending to children's strategies, (b) interpreting children's understandings, and (c) deciding how to respond on the basis of children's understandings. Our becoming aware of student strategies and eliciting the phases of the strategies agree with the first two skills of the *Noticing* model. As in the *Community of Practice* framework (Heinze & Procter, 2004; Kahan, 2004), communication and interaction among teachers are critical in this model. In both frameworks, the final phase is devoted to formulating a response and/or reaction as a result of the analysis of student understanding and thinking. Each phase should be followed by an exchange of ideas and in-depth reflective discussion for collective inquiry (Kazemi & Franke, 2004).

Methods

This study was part of a year-long professional development program funded by the Improving Teacher Quality Program. The main content foci of this externally funded PD were "Number Sense and Operations" and "Algebra". The PD program was designed using a hybrid model. Teachers participated in: monthly face-to-face workshops meetings; five 6-hour summer sessions; four 6-hour fall sessions; and four 6-hour spring sessions. Between sessions, participants established online professional learning communities by communicating with each other and the instructors. These virtual learning included textbook reading discussions, implementing student-centered discovery lessons and reflecting on them, sharing instructional ideas, providing comments on other participants' lessons, watching a best practice lesson and reflecting on it, and reviewing and evaluating web-based mathematical instructional resources.

This Task Design and Analysis activity was among several year-long PD activities implemented in the fall, connecting professional learning and practice. The purpose and detailed explanation of the Task Design and Analysis PD activity are summarized in Table 2. Thirty inservice teachers participated in the program, and their demographic information is summarized in Table 1.

Table 1
Demographic Information of Participants (n=30)

		N
Gender	Female	28
	Male	2
Race/Ethnicity	White, non-Hispanic	30
Position	Teacher	20
	Special Education, Resource or Inclusion Teacher	7
	Building Coach	3
Grade Level Taught	Primary (K-3)	13
	Intermediate (4-6)	16
	Middle (7-8)	1
Classroom Type	Self-contained class (for all or most subjects)	17
	Math only	3
	Math and Science	2
	Other or Multi-Subject combinations	8

Data Collection

The mathematics problem used in this study can be found in *Comparing Quantities, Algebra* of the *Mathematics in Context* series (Kindt et al., 2010). The question provided the total price for two umbrellas and one hat ($80) and the total price of one umbrella and two hats ($76) and asked for the price of one hat and the price of one umbrella. Before the *Comparing Quantities* section, students learned two strategies for solving similar problems: (1) exchanging and (2) making a combination chart and using number patterns found in the chart (van Reeuwijk, 1995). Students were expected to "apply the strategy of exchanging to solve problems involving the method of fair exchange." (p. 16)

For algebra teachers and students in grades 6 and higher, this problem involves two variables and two equations. According to Meyer (1997), 6^{th} graders can solve this problem without using equations. If so, how about students in earlier grades? Can they also solve the problem and what strategies would they be using? How will these strategies differ from those used by students exposed to pre-algebra and algebraic concepts? And how about the strategies used by teachers to solve the problem; will they differ depending on assigned teaching grade levels?

Data were collected in each phase of the Theoretical Framework of the study (Figure 1). A detailed description of the task design and analysis activity and data are summarized in Table 2.

- Understanding teacher knowledge:
  (1) Understanding the original task: As part of the initial phases of the task design and analysis project, the teachers were asked to solve the problem. Teachers worked individually for 5 - 15 minutes, shared their solutions in the small group and developed a rubric, graded each other's work using the rubric, and then discussed the whole process with the entire

class. There were no pre-instructions similar to those given prior to the Hats and Umbrellas problem in the PD.

- Understanding students' strategies:
  (2) Setting up a task: After the first phase, teachers were asked to use the task in their respective classes and collect the students' solutions.
  (3) Becoming aware of student strategies: Teachers were asked to analyze students' strategies by using a commercially designed rubric focusing on students' problem solving skills. Teachers shared their grading criteria with other project teachers. Teachers were also asked to select the three most interesting solutions (regardless the accuracy) provided by their students, along with their rationale behind their selection. A total of 90 solutions (30 teachers providing 3 solutions each) were thus selected for consideration.
  (4) Eliciting student ideas: The project team reviewed 90 selected student strategies and chose 15 solutions for the activity of the next phase: developing efficient and high level thought-provoking questions for individual students. Teachers were asked to pose appropriate questions for each student to respond to their needs and to improve their learning outcomes.
- Developing/revising a task:
  (5) Developing an instructional trajectory: Teachers were asked to reflect on the entire cycle of the task design and analysis PD activity and develop two revisions to teach the same mathematical concept as the original task.

Table 2
Professional Development (PD) Activity and Data

Purpose	Phase	PD activity	Data collected
Understanding teachers' content knowledge	1.Understanding the original task	Face-to-face PD workshop (Sept.) 1) Solve the task individually 2) Group discussion (small & class)	Teachers' solution
Understanding students' strategies	2. Setting up a task	Classroom (Sept. - Oct.): Teachers use the task in their own teaching.	Students' solutions
	3. Becoming aware of students' strategies	Off campus (Oct. - Nov.) 1) Grade students' work 2) Analyze students' strategies 3) Select three most interesting students' solutions 4) Rationalize for selecting the three solutions.	Graded scores Selections of students' solutions Rationale for selecting three solutions
	4. Eliciting students' ideas	Face-to-face PD workshop (Nov.) 1) Share students solutions and grades 2) Develop questions for each student	Questions with reasoning for supporting each question
Developing/ Revising a task	5. Developing an instructional trajectory	Off campus (Nov. – Dec.) 1) Reflect on the PD activity 2) Develop two revisions	Reflection paper Revised tasks

Data Analysis

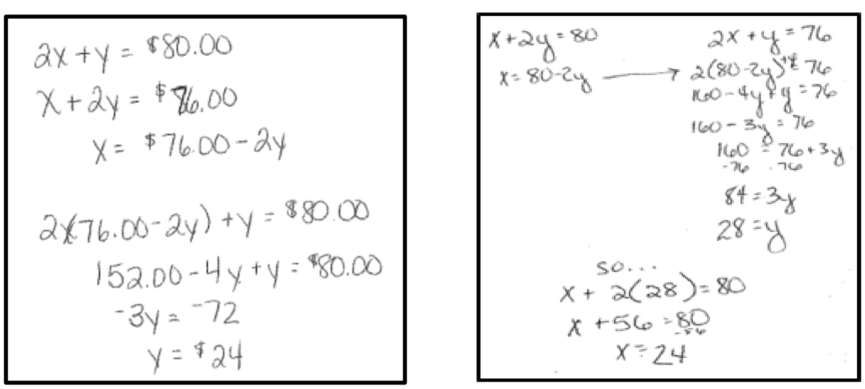
Due to the nature of the qualitative data and in order to investigate emerging themes (Miles & Huberman, 1994), the analysis was based on categorizing. Teachers' solutions were analyzed for common codes to create patterns. Instead of sharing all their student data with the PD group, teachers were asked to select the three most interesting student solutions, and to provide the rationale they used for selection. Polya's problem solving model was used as a framework to analyze to teachers' selections. The teachers' revisions (revised tasks) were then analyzed for common codes to look for patterns. Rittle-Johnson and Koedinger (2005) state that "well-structured knowledge requires that people integrate their contextual, conceptual and procedure knowledge in a domain" (p. 313). The design of a learning environment, therefore, scaffolds all these knowledge types. During the data analysis of revised tasks, we were guided by the idea of "well-structured knowledge" provided by Rittle-Johnson and Koedinger (2005), and used contextual, conceptual and procedural changes, as a framework. Throughout the data analysis process, common codes and patterns were tallied (Miles & Huberman, 1994) and percentages of participants sharing similar revisions were calculated from those tallies. Each investigator coded the data independently. The project team then conducted a co-coding session in order to resolve conflicts and create the common codes where the individual codes were compared. When the investigators coded an item differently, they negotiated an agreement through discussion. Investigators and data triangulation ensured the trustworthiness of this study (Denzin & Lincoln, 1994).

Results

Teacher Knowledge of Mathematics: Understanding the Task

The in-service teachers employed two different approaches in their work on the Hats and Umbrellas problem: symbolic manipulation and the guess and check method. Ten teachers started the problem by trying out some numbers. The guess and check method was not successful for five of those teachers, but it may have helped them gain insight into the possible range of solutions. Among the five who did obtain the correct answer, one teacher was able to intuit that an umbrella costs $4 more than a hat. The teacher shared that this informed his or her guess and check process.

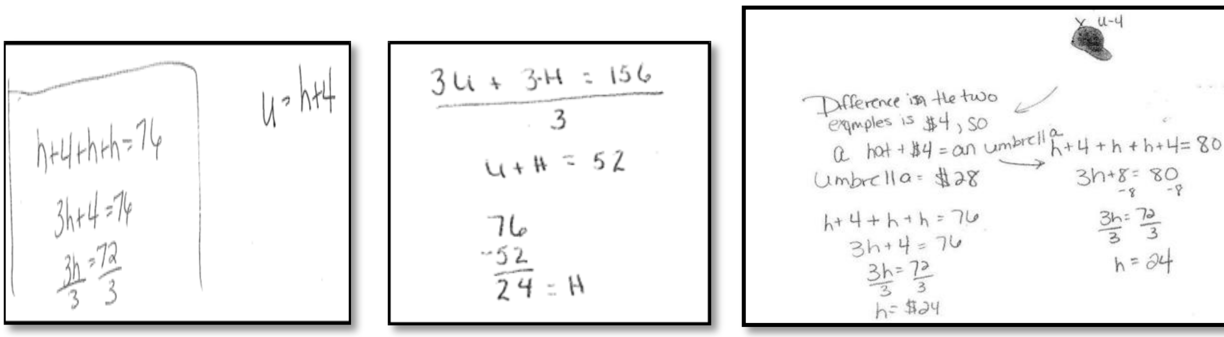
The symbolic method, used by 25 (89%) teachers, was the major primary or secondary method used to solve this problem. Only 3 teachers relied solely on the guess and check method. However, teachers used the symbolic approach in different ways. Thirteen set up a system of equations and all thirteen solved it using the substitution method (see Figure 2). The rest used more than just symbolic manipulation.



a b

Figure 2. Symbolic approach using substitution.

Ten teachers used the idea of exchanging, which van Reeuwijk (1995) called *reasoning through exchanging*. Looking at the Hats and Umbrellas problem, one can see that an umbrella costs (exactly \$4) more than a hat. Five teachers used the equation u= h+4 and solved one of the equations for the price for one hat or the price of one umbrella. So, exchanging the umbrella in the second equation would lead to three hats and four dollars, which can then be easily solved for the price of one hat. The other five teachers used reasoning with exchanging slightly differently. They were able to conclude that one hat and one umbrella together would cost \$52, by adding the two initial equations and dividing by 3. Then they exchanged the total for 1 hat and 1 umbrella in one of the equations to obtain the price for 1 hat or 1 umbrella (see Figure 3).



a b c

Figure 3. Reasoning with exchanging.

Two of the thirteen teachers who solely used the symbolic approach, used this idea of exchanging symbolically. They were able to see that there was a \$4 difference between the two equations and used it to set up a new equation as seen in Figure 4 to obtain that u= h+4 solely symbolically. Then they solved one of the equations using this newly constructed equation.

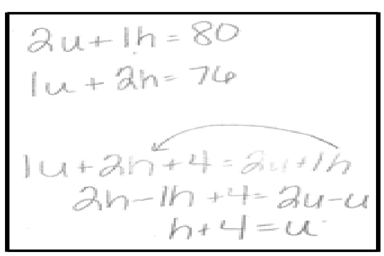


Figure 4. Symbolic exchanging.

Teachers' Understanding of Student Cognitive Activity: Understanding Student Strategies

The teachers graded the students' solutions in five areas: Mathematics Language, Representation, Presentation, Problem Solving, and Mathematical Accuracy. The teachers were asked to rate a student's weak areas based on how they graded the student's solution. Figure 5 summarizes the findings of the grading outcomes. Approximately one-quarter of the teachers reported mathematical language as an area weak of weakness, while mathematical accuracy seemed to have the least concern to teachers. During the grading process, teachers engaged in a lengthy discussion about the difference between presentation and representation.

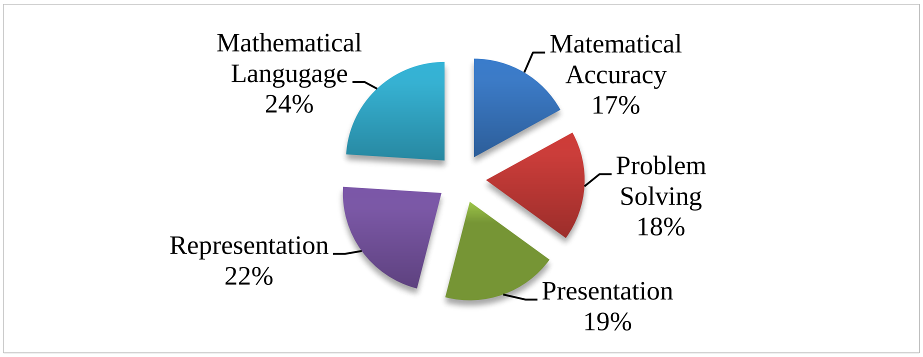


Figure 5. Areas of student weakness.

Teachers also analyzed students' strategies and developed an understanding of the students' cognitive activity. Students' strategies that teachers noticed and valued are understanding the problem, using prior knowledge, using heuristics, being able to use algebraic representations, and checking and looking back a solution.

As part of this phase, teachers were asked to select the three most interesting solutions after grading the student work and analyzing the solution strategies. All teachers valued the basic steps of a problem solving process: understanding the problem, working on the solution using heuristics, and looking back. *Understanding the problem* was one of the ideas highlighted by almost 50% of teachers. Teachers were pleased to see a student realize that they needed to work with both equations simultaneously, or that the price of one hat or one umbrella should be the same for both equations. If a student neglected this fact, then the student could not solve the problem correctly, and teachers saw this as a major deficiency. One teacher pointed out that a certain student (see an example in Figure 6) did not understand "the directions (hat/umbrella cost should remain constant throughout)" when working on this problem.

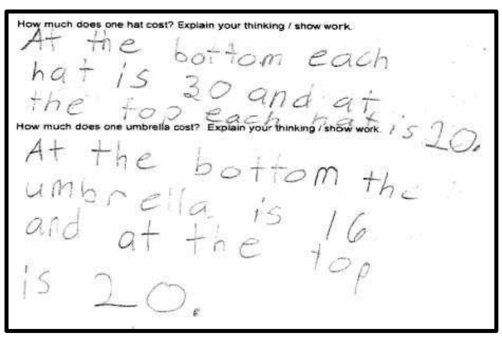


Figure 6. Assigning different prices for items.

Teachers were gratified to see some students *bring their previous knowledge* such as operational skills to the solution process. Approximately 30% of the teachers valued that students were able to carry on computations correctly. The teachers (almost 43%) noticed that some students divided a total price by 3 when there were 3 items in each situation of the problem. Being able to take into account both equations and analyzing their relationship leads to the conclusion that there is a $4 difference between the price of one hat and the price of one umbrella. Whether or not a student was able to make this conclusion was mentioned by approximately 40% of the teachers. A teacher shared that she/he chose the student work shown in Figure 7, because the student "found that an umbrella costs $4 more than a hat. He then used reasoning to $76-$4 for the umbrella in row 2, and then divided the $72 left by 3 items."

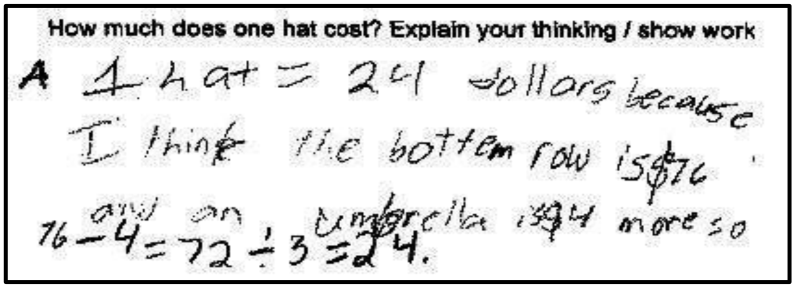


Figure 7. Price difference.

Teachers valued *the heuristics* used by students during the problem solving process. The most commonly used heuristic was the guess and check method, and approximately 74% of teachers discussed this as one of the reasons for selecting interesting student work. Some of the teachers highlighted that they especially appreciated a student's use of additional strategies to make an informed guess. In one example, a student noticed the $4 difference, which subsequently affected their guess and check process. Other students, as described by one of the teachers, informed their guess and check process after dividing 80 or 76 by three.

Being able to use algebraic representations was mentioned by five teachers. A teacher explained that even though some students did not pursue the algebraic approach to solve the problem, constructing an algebraic representation was important. Two teachers mentioned the similarity between their own solutions and the student solutions they selected. Finally, even though checking a solution should happen naturally during the guess and check method, almost 22% of the teachers highlighted the importance of checking a solution/looking back in the problem solving process.

Teacher Ability to Develop High Quality Tasks: Revision Pattern

When analyzing the changes that teachers made to the task, the main categories for revisions were contextual changes, conceptual changes, procedural changes, and format related changes. Teachers made two different types of contextual changes (32%) to the original problem: enhancing the story by giving more background to the story and changing the objects in the story. Conceptual changes (38%) were made by changing the pattern of objects, key questions, mathematical signs, comparisons, or dollar amounts. Teachers also revised the task to help students at the procedural level (23%) by adjusting the numbers in the problem or asking students to create their own combination before or after solving the problem. Some teachers changed the format (7%) of the task by providing a chart to help students organize their thoughts and keep track of their guesses or providing boxes to enter specific answers. Figure 8 summarizes details within each revision area. Teachers used different objects and different numbers to revise the original task. Changing the story (direction) and pattern were also common ways of task revision.

The original problem had a pattern umbrella, hat, umbrella (uhu) and uhh. As can be seen in Figure 8, revisions to the pattern of the objects were the most commonly adopted approach in revising the task conceptually. Some early grade teachers provided patterns with only one object, and other changes focused on making sure that students would understand that a hat in the first pattern and the second pattern would be same price (see Figure 9). Also, another way to revise the task conceptually was to change key questions. For example, instead of providing the total prices of multiple items, revised tasks provide one of two items and total prices and ask students to find the price of the other item. Others asked additional questions such as the price after tax or discount. The last conceptual changes were about representation of the problem. Some teachers added plus signs or equal signs between objects with the aim of helping students transfer the problem from the contextual world to the mathematical world. Teachers also revised the task to help students at the procedural level by adjusting the numbers in the problem, or asked students to create their own combinations before or after solving the problem.

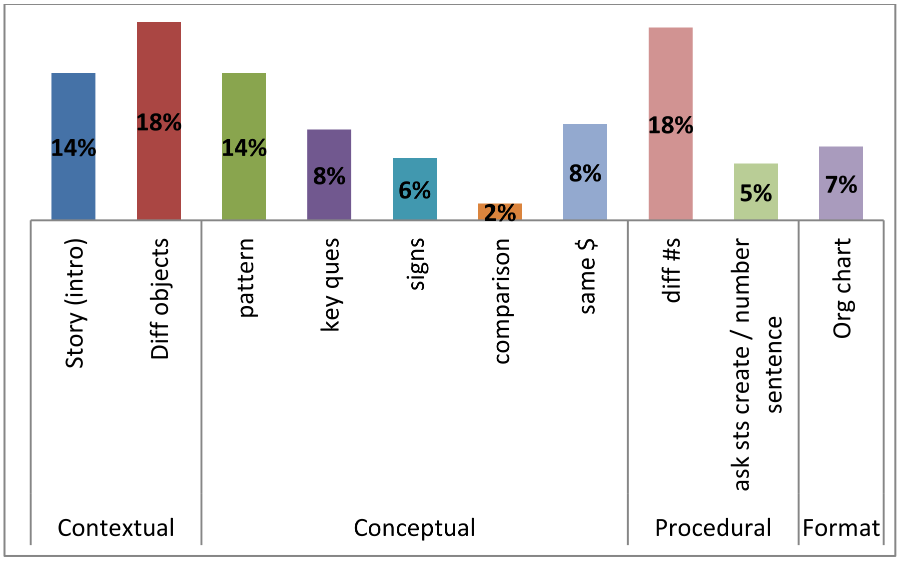


Figure 8. Task revision pattern.

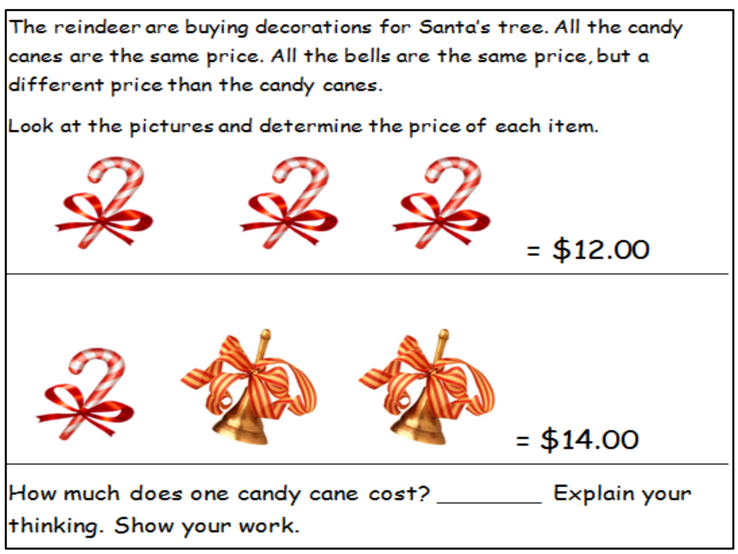


Figure 9. Task revision example.

The findings of the study suggest that teachers attempted to adjust the level of difficulty of the task based on their students' knowledge and tried to engage students through contextual changes. For example, some teachers changed the items in the story (e.g. Christmas items due to timing of the academic year as in Figure 9 or toys) to arouse the interest of students. Other teachers changed the story altogether. However, there was not enough evidence to claim, according to Swan's (2007) criteria, that the teachers of this project succeeded in developing high quality tasks. Developing a cognitively demanding quality task requires the teacher to be exposed to a number of exemplary tasks, not just the practice of developing and analyzing a single task.

Teachers' End-of-Program Reflection

Teachers' reflections on their experience in the task design and analysis activity were categorized in three major areas (Figure 10): (1) reflecting on self/practice (light blue items), (2) reflecting on the overall professional development experience (orange items), and (3) connecting the learning from the professional development program to own instruction (purple items).

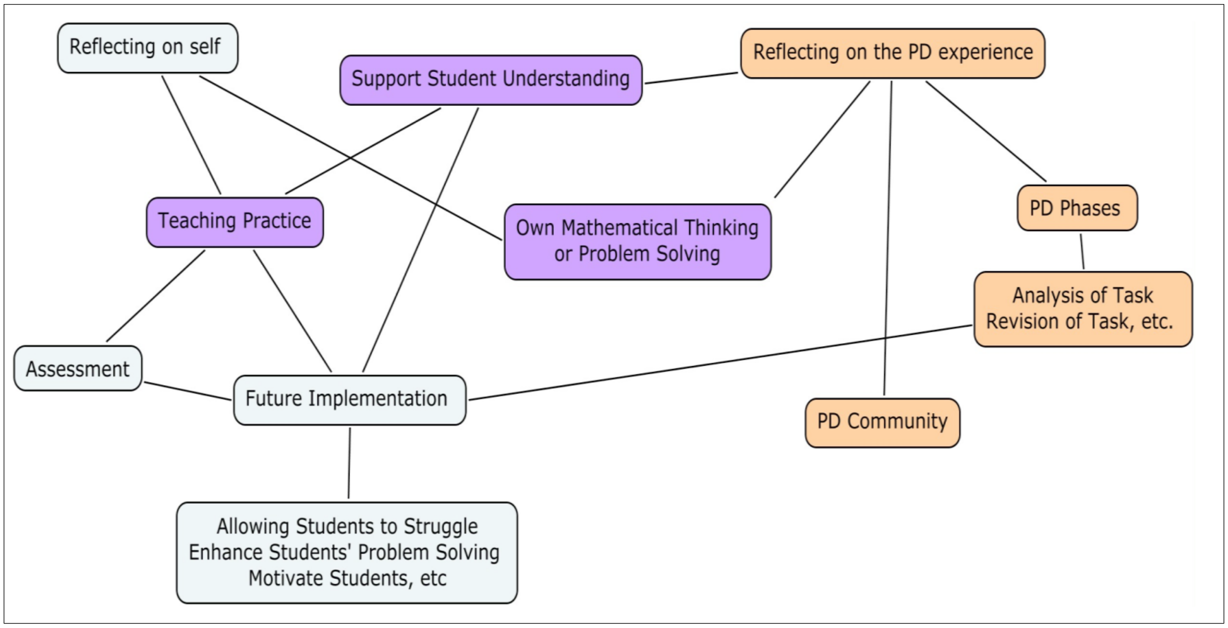


Figure 10. Teacher reflections on the task design and analysis experience.

The issues teachers discussed in relation to the PD experience were mainly about different phases of PD such as analysis of task, analysis of student work, and revision of the task. Teachers also mentioned that they enjoyed working on these tasks together as a group and building a community.

> I couldn't wait to share my ideas with my group and I also had an Ah ha moment when listening to my classmates approach.
>
> The "big idea" that I have learned is that working together makes us better educators. The most beneficial aspect for me was the discussion that we held as educators when looking at the papers and discussing the results. Our small group noticed details of each student's work that I may not have as an individual.

Many of them also reflected on themselves as a "doer of mathematics" and "teacher of mathematics." When focusing on the *doer of mathematics* perspective, they discussed how their mathematical thinking and mathematical problem solving were influenced as a result of the PD.

> Before working on modifying this problem, I thought that I was proficient at amending problems for students' and teachers' benefit. Amending this problem caused me to grow professionally. More than ever, I found myself thinking deeply about the math; I worked problems algebraically; I worked with tables in a guess and check format; and I struggled to make this problem better not easier!

When they were reflecting as *teacher of mathematics,* their reflections centered on their teaching practice. For instance, how they can enhance their future implementation to support students' understanding or how student knowledge could be assessed more efficiently.

> I have learned to look beyond finding whether an answer is simply right or wrong. I have always tried to give partial credit for showing the thinking process and the logic involved but the rubric we used made it much easier and I had never seen anything like it used in mathematics before. It made it easier to identify where students' misconceptions were and whether they truly understood mathematical processes involved in problem-solving.

Teachers' reflections indicated that they want to connect the learning experience gained from the PD to improve instruction in the area of *ways to support student learning* by better understandings students' strategies and allowing them to struggle productively.

> One big idea that I have learned is that I need to not simply call on students for the answer, but for the explanation of how they achieved their answer. Many students think that math is just calculating numbers, but it is so much more. Students today are not always very good at explaining how they find answers. I would love to have students write a "How To" speech about solving a problem so each of them could see all of the steps it takes to explain their own thought processes. The cool thing about that would be that everyone thinks a little bit differently, so hopefully we would have the same end result, but several different strategies to find it.
>
> I am trying to stretch my questioning ability to encourage deeper thinking. It is tempting to use straight forward questioning that is easy to write and quick to answer. However, these questions do not lead to persisting through a multiple step problem and creates an apathetic learner that looks only for key words and does not read for understanding.

Discussion

The quality of a task depends on its features and cognitive demands, the thinking processes used while performing the task. The level of cognitive demands can range from memorization to the use of procedures and algorithms to complex thinking and reasoning strategies involving conjecturing, justifying, or interpreting (Henningsen & Stein, 1997). Mathematics educators have identified tasks as represented in curricular/instructional materials or as set up by the teacher in a classroom. Tasks set up by a teacher are influenced by the teacher's goals, subject matter knowledge, and knowledge of students and student thinking and understanding. By reflecting on and taking into consideration students' thinking and understanding, the teacher can provide a task that better improves/advances students' thinking. To foster a more active implementation of tasks, teachers should encourage students to use various strategies, use multiple representations, and provide justifications.

This study had two main goals: (1) to develop a conceptual framework for a professional development (PD) program that uses student work to enhance a teacher's professional growth; and (2) to provide an analytic framework within which teachers and teacher educators analyze that student work.

Before we can start to think about how to help students learn and do mathematics, we need to be able to understand how a student perceives a task and thinks about its solution. In this study, we designed a professional development activity involving task analysis and redesigning a common mathematics task. Teachers engaged in 5 phases (shown in Figure 1: Theoretical Framework) to complete this activity. The purpose of the activity was to help teachers improve their teaching as a result of analyzing student work, understanding student's strategies, and learning to develop a high cognitive task. In the first stage of the activity, the teachers worked on the problem by themselves. The main aim of this stage was to focus on the content of the problem, hence the mathematical knowledge of the teachers. Participants, as doers of mathematics, first solved the problem individually, and then as a group. Even though the majority of teachers (89%) solved this problem algebraically, they knew not to expect the same from their students. Before using the task with their students, the teachers reflected on the mathematics of the problem. This was crucial to the discussion on what to anticipate from their students and how to prepare the task for their own students and keeping in mind the importance of making sure that the students justify their solution and explain their strategies and thinking.

Once the teachers administered the problem in their classrooms, they collected the student work for further analysis. The analytical framework for the analysis of student work included: (1) grading student work, (2) analysis of student strategies, (3) selecting student work, and (4) providing rationale for their selections. Most teachers' authentic classroom work is limited to just the first level of this framework, simply grading the students' work. In the second stage of the analytical framework, the teachers took a closer look at their students' work, by analyzing their students' strategies. In the third stage, teachers selected "interesting" solutions and in the fourth stage they were asked to explain why they selected a particular solution. Two main overarching categories for selecting a solution were: choosing a successful student solution and an unsuccessful solution.

While successful solutions showcased teachers' accomplishments in their classrooms, unsuccessful solutions helped them to think about their next instructional steps. Teachers valued that their students were able to use their previous knowledge, make correct computations or successfully used the informed guess and check method. However, most teachers also concluded that students had difficulties understanding the problem or analyzing the context of the problem in order to reach a solution; for instance having the same price for a hat in both situations. As a natural next step, specific questions were developed to be posed to students and scaffold their high level thinking. Such follow up questions can lead to a more effective instructional trajectory.

Teachers see a multitude of student work on a daily basis. Most of the time, the major goal is to quickly assign a grade to the homework or an exam without reflecting too much on student thinking or use such assessment materials as foundational information for the following lesson. So many questions appear in a homework assignment or exam, that it would be unrealistic to expect teachers to analyze each of them in detail. Instead, we suggest that if teachers assigned one or two rich tasks, which can be approached and solved in multiple ways, to students and carefully analyze their solutions, they would gain more information about their students' thinking and inform their instruction more efficiently.

References

Denzin, N. K., & Lincoln, Y. S. (Eds.) (1994). *Handbook of qualitative research*. Thousand Oaks, CA: SAGE Publications.

Heinze, A., & Procter, C. (2004). *Communication – A challenge and an enabler for facilitating Blended Learning community*. Salford, UK: University of Salford.

Henningsen, M., & Stein, M. K. (1997). Mathematical tasks and student cognition: Classroom-based factors that support and inhibit high-level mathematical thinking and reasoning. *Journal for Research in Mathematics Education, 28*(5), 524-549. doi:10.2307/749690

Jacobs, V. R., Lamb, L. L. C., & Philipp, R. A. (2010). Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education, 41*(2), 169-202.

Kahan, S. (2004). Etienne Wenger on communities of practice: Engagement, identity & innovation. *The Journal of Association Leadership, 26*. Retrieved from https://www.shareweb.ch/site/Learning-and-Networking/sdc_km_tools/Documents/Etienne-Wenger-on-Communities-of-Practice-2005.pdf

Kazemi, E., & Franke, M. L. (2004). Teacher learning in mathematics: Using student work to promote collective inquiry. *Journal of Mathematics Teacher Education, 7*(3), 203–235. doi:10.1023/b:jmte.0000033084.26326.19

Kindt, M., Abels, M., Dekker, T., Meyer, M. R., Pligge, M. A., & Burrill, G. (2010). Comparing quantities. In Wisconsin Center for Education Research & Freudenthal Institute (Eds.), *Mathematics in context* series. Chicago, IL: Encyclopædia Britannica, Inc. Retrieved from http://mathincontext.eb.com/content/books/students/Comparing_Quantities_9781593399177txt_LR.pdf

Krebs, A. S. (2005). Analyzing student works as a professional development activity. *School Science and Mathematics, 105*(8), 402-411. doi: 10.1111/j.1949-8594.2005.tb18060.x

Lee, H. J., & Özgün-Koca, S. A. (2013). A professional development model for task design and analysis. In C. A. Shoniregun & G. A. Akmayeva (Eds.), *Proceedings of the London International Conference on Education (LICE-2013)* (pp. 46-50). Infonomics Society: London, UK.

Lee, H. J., & Özgün-Koca, S. A. (2014, April). *Task design and analysis: A mean to increase teacher growth*. Paper presented at the 2014 Annual Meeting of the American Educational Research Association (AERA), Philadelphia, PA.

Meyer, M. R. (1997). Mathematics in context: The gates to mathematics for all at the middle level. *NASSP Bulletin, 81*(586), 53-59. doi: 10.1177/019263659708158606

Miles, M. B., & Huberman, A. M. (1994). *Qualitative data analysis: An expanded sourcebook* (2nd ed.). Thousand Oaks, CA: SAGE Publications.

Namiki, R., & Shimizu, Y. (2012, July). *On the nature of mathematical tasks in the sequence of lessons*. Paper presented at the 12th International Congress on Mathematical Education, Seoul, South Korea.

Rittle-Johnson, B., & Koedinger, K. R. (2005). Designing knowledge scaffolds to support mathematical problem solving. *Cognition and Instruction, 23*(3), 313-349. doi:10.1207/s1532690xci2303_1

Stein, M. K., Grover, B. W., & Henningsen, M. (1996). Building student capacity for mathematical thinking and reasoning: An analysis of mathematical tasks used in reform classrooms. *American Educational Research Journal, 33*(2), 455-488. doi:10.3102/00028312033002455

Swan, M. (2007). The impact of task-based professional development on teachers' practices and beliefs: A design research study. *Journal of Mathematics Teacher Education*, *10*(4-6), 217-237. doi:10.1007/s10857-007-9038-8

Thompson, P. W., Carlson, M. P., & Silverman, J. (2007). The design of tasks in support of teachers' development of coherent mathematical meanings. *Journal of Mathematics Teacher Education, 10*(4-6), 415–432. doi:10.1007/s10857-007-9054-8

van Reeuwijk, M. (1995). *The role of realistic situations in developing tools for solving systems of equations*. Paper presented at the annual conference of the American Educational Research Association, San Francisco, CA. Retrieved from http://www.fisme.science.uu.nl/publicaties/literatuur/3781.pdf

Author Notes

Hea-Jin Lee, Ph.D.
The Ohio State University at Lima, Department of Teaching and Learning
4240 Campus Dr. Lima, OH. 45804
Lee.1129@osu.edu

Dr. Hea-Jin Lee is Associate Professor of mathematics education at The Ohio State University. Her research addresses: improving reflective thinking and practice; assessing professional growth; designing teacher-needs based professional development programs; analyzing the effectiveness of technology-enabled teacher professional development; and teaching mathematics to children at various levels.

S. Asli Özgün-Koca, Ph.D.
Wayne State University
5425 Gullen Mall, 249 Education Building, Detroit, MI. 48202
aokoca@wayne.edu

Dr. S. Asli Özgün-Koca is Associate Professor of mathematics education in the College of Education of Wayne State University with special expertise in the use of technology to enhance middle school and high school mathematics education. Her major area of research interest is to study the process by which students use graphing calculators, other handheld devices and software, and to develop curriculum materials with authentic contexts for middle school students.

Current Issues in Education
Mary Lou Fulton Teachers College • Arizona State University
PO Box 37100, Phoenix, AZ 85069, USA

Volume 19, Issue 2 | August 15, 2016 | ISSN 1099-839X